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Can one hear the shape of a saturation patch?

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Short title: CAN ONE HEAR THE SHAPE OF A SATURATION PATCH?

Abstract. The theory of the acoustics of patchy-saturation in porous media is used to analyze experimental data on wave velocity and attenuation in partially water saturated limestones. It is demonstrated that the theory can be used to deduce the value of V/A , the ratio of the volume to area of the water patch, and l_f , the Poisson size of the water patch. One can “hear” the shape of a patch if the properties of the rock and the measurement frequencies are such as to satisfy the specific requirements for the validity of the theory.

The Biot theory [Biot, 1954] can be used to describe the acoustic properties of an elastic porous solid fully-saturated with a compressible Newtonian fluid. (For a review see [Johnson, 1986] and references therein.) The input parameters of the theory are directly measurable on a given sample by independent means. The success of the theory in predicting the dispersion and the attenuation of the fast compressional, slow compressional, and shear modes has been demonstrated in a number of relevant experiments ([Johnson, 1994] and references therein).

Johnson [2001] has extended this theory to the case of an elastic porous frame containing two different fluids filling the entire frame in non-mixing “patches” (i.e. at every point of the sample, the

frame is fully-saturated with one of the two fluids). The theory is a simplification and generalization of ideas first suggested by [White, 1975] and subsequently developed by others. Here, the basic mechanism for attenuation/dispersion is that when the sample is compressed the pore pressure in the patch containing the stiffer fluid tries to equilibrate with that in the more compressible patch by means of fluid flow from the one region to the other. The effect is maximized if one of the fluids (e.g. air) is much more compressible than the other (e.g. water). In this Letter we use this theory of the Acoustics of Patchy-Saturation (APS) to analyze measurements by *Cadoret et al.* [1993, 1995, 1998] of wave velocities and attenuation in partially water saturated limestones. Specifically, as a function of fractional water-saturation, S , we deduce the values of V/A , the ratio of the volume of the water patch to the bounding area, and of the Poisson size, l_f , a different, but equally well-defined, measure of patch size. (In the special case that the water patch is a d -dimensional hypersphere of radius R , for example, one has $V/A = R/d$ and $l_f = R/\sqrt{d(d+2)}$.) It is in this sense that one can “hear” information about the size and the shape of the patch; the wavelength of the slow compressional wave is being used as the yardstick. As far as we are aware, there is no other technique that could allow one to make this kind of a deduction.

The APS theory makes several simplifying assumptions which ultimately restrict the kinds of samples which can be analyzed by it: (1) The Biot theory is presumed to be the only significant mechanism for attenuation/dispersion. This assumption rules out the applicability of the APS theory to sandstones, for example, which are known to be dominated by so-called microscopic squirt mechanisms. (2) The measurement frequencies are so low that within each patch the Biot theory is strictly in the low-frequency limit: $\omega \ll \omega_B$, where

$$\omega_B = \frac{\eta\phi}{k\rho_f\alpha_\infty}, \quad (1)$$

in terms of the fluid viscosity, η , the porosity, ϕ , the permeability, k , the fluid density, ρ_f , and the tortuosity of the pore space, α_∞ . For samples of high permeability, this restricts the applicability of the theory to rather low frequencies. (3) The frequencies are low enough that the wavelengths are large

compared to a characteristic patch size, L : $\omega \ll \omega_x$ where

$$\omega_x = \frac{2\pi V_{\text{sh}}}{L}. \quad (2)$$

V_{sh} is the velocity of the shear wave. For samples with large patch sizes, this also restricts the validity of the theory to rather low frequencies. (4) Capillary effects, or rather the change in capillary forces induced by the acoustic wave, are neglected. Capillary effects clearly become more significant for smaller pore sizes, i.e. for samples of lower permeability. Moreover, if there is a very broad distribution of pore sizes, capillary effects can lead to a very ramified, “fractal”, structure for the patch geometry. (5) The sample is macroscopically homogeneous, except for the saturation patches.

Subject to the approximate validity of these assumptions, the APS theory describes how the dynamic bulk modulus, $\tilde{K}(\omega)$, crosses over from the Biot-Gassmann-Woods result at low frequencies to the Biot-Gassmann-Hill result at high:

$$\tilde{K}(\omega) = K_{\text{BGH}} - \frac{K_{\text{BGH}} - K_{\text{BGW}}}{1 - \zeta + \zeta \sqrt{1 - \omega\tau/\zeta^2}}. \quad (3)$$

The low and the high frequency limits, K_{BGW} and K_{BGH} respectively, depend upon the usual Biot parameters, as well as the value of the saturation, but do *not* depend upon the patch geometry. We note for future use that if the gas phase is taken to be infinitely compressible, then $K_{\text{BGW}}(S) \equiv K_b$ (the bulk modulus of the solid frame) for all values of saturation, $S < 1$. The parameters ζ and τ *do* explicitly depend upon the two patch geometry parameters V/A and l_f , as well as the usual Biot parameters, cf. Eqs. (44) and (45) of [Johnson, 2001]. It was demonstrated in [Johnson, 2001] that Eq. (3) gives a very good approximation for the dynamic modulus of patches that are slabs (d=1), cylinders (d=2), spheres (d=3), or hollow spheres. There are no adjustable parameters in the theory. In the following, we assume that one knows all the Biot parameters for the system and we investigate the effects of the partial saturation. Our procedure is to fit the relevant experimental data with Eq. 3, thereby extracting values for ζ and τ . With the values of ζ and τ one can easily solve for V/A and l_f , for each value of the saturation. In order to demonstrate this, we analyze the experimental findings of Cadoret *et al.* [1993, 1995, 1998].

We need data on samples which satisfy the aforementioned assumptions and for which acoustic measurements were taken at widely separated frequencies. As far as we are aware the best data for our purposes is that of Cadoret who performed two sets of experiments on eight types of limestone (see Table I). This suite of samples spans a wide range of permeabilities, even though the porosities are all quite high. First, he measured the velocity and attenuation of extensional and shear waves using the resonant bar technique. The samples were obtained by drilling out (parallel to the rock bedding plane) a cylindrical rod with a length of 110 cm and a diameter of 8 cm. The lowest resonant harmonic frequencies are in the range 1-2 kHz, as shown in Table I. Second, he performed ultrasonic measurements of the compressional wave velocity at 50 and 100 kHz. (He also reported measurements at 0.5 and 1.0 MHz, which we do not use here.) In both sets of experiments, the samples were desaturated by drying technique. By means of a careful analysis of these results, as well as those taken on samples which were desaturated by a depressurization technique, *Cadoret et al.* [1993, 1995, 1998] make a convincing case that the dominant mechanism for dispersion and attenuation in the sample desaturated by drying is the acoustically induced flow from one patch to the other.

Table I

Our first step is to use the sonic data to deduce values for the complex-valued bulk modulus $\tilde{K}(\omega)$ at the measurement frequency. Then we use Eq. (3) to find V/A and l_f as described above. Fig. 1 shows the bulk modulus dispersion and attenuation found from the resonant bar data of Cadoret for four kinds of limestone. Our convention is

Fig. 1

$$\tilde{K}(\omega) = \Re(\tilde{K})(\omega) \left(1 - \frac{i}{Q_K(\omega)} \right). \quad (4)$$

We find K_b from the $\Re(K)$ data by finding the “flat” region of its saturation dependence at $S < 0.5$. It is marked by the dotted line. (The shear modulus N is found analogously from $\Re(N)$ data which we do not show here.) The value of K_b found this way is different from the bulk modulus of the completely dried frame (as one can see from the top plot) since a very small amount of water leads to a noticeable softening of the sample. Since the porosity, ϕ , the bulk and shear frame moduli, K_b , N and the solid modulus K_s (≈ 69 GPa for limestone) are now known, it is straightforward to calculate

K_{BGH} as a function of saturation. This is plotted as a dashed line. The dotted line on the second plot of Fig. 1 shows the “flat” region of the saturation dependence of the attenuation $1/Q_K$ at $S < 0.5$. The mechanism of this attenuation is clearly different from and additional to the patchy-saturation mechanism. In our analysis we subtract off this contribution to $1/Q_K$ before solving for V/A and l_f .

In the left plots of Fig. 1, the data lie between the high and low frequency limits for the Estailades and Lavoux RG limestones. This is expected from the theory. On the other hand, the theory breaks down in the case of the other two samples. We do the same analysis for the rest of the limestones listed in Table I. We separate them into three categories, corresponding to low, medium and high permeability. All of these limestones have similar porosities and all have calcite mineralogy. We find that the APS theory is mostly applicable to the rocks with medium permeability (Lavoux RG, Estailades and Ménerbes). In the following, we analyze data of Fig. 1 for the Estailades and Lavoux RG limestones and then discuss why the theory does not work in the case of the high and low permeability samples.

As discussed in [Johnson, 2001], the APS theory can apply if the surface of the sample is either sealed or open as long as the gas phase may be considered to be infinitely compressible. Since the experimental procedure for de-saturating the samples was different in the two cases, the parameters V/A and l_f may also be expected to be different in the two cases, for a given value of S . In the sealed interface measurements, the boundary of the fully water saturated rock was first jacketed and then the sample was dried by making a few small open holes. This leads to a more heterogeneous patch structure in the sample; while completely open surface drying leads to air patches more or less contiguous throughout the system, drying with only a few open holes makes air saturate the sample preferentially near the holes, which are far apart. The latter situation leads to larger patch sizes and, as a result, a tighter limitation on the theory through Eq. (2). That is, the relevant value of L may be comparable to the length of the rod, rather than to the radius. We think the case of a closed interface lies beyond the applicability of the APS theory as ω exceeds ω_x and we have not pursued it any further.

In Table I we show the Biot crossover frequency ω_B as well as ω_x . The latter is evaluated by setting $L = 2R$ in Eq. (2). We expect this to be an overestimate of the patch sizes in the case of open

interface, but in the case of closed interface with high saturation values, this can be an underestimate, as discussed above. (See *Cadoret et al.* [1993, 1995, 1998] for images of the computerized tomography scans of fluid distribution within the rock.)

Fig. 2 shows characteristic patch sizes extracted from Cadoret's measurements on the open interface. The dotted lines on the plot indicate the value $R/2$ ($R/\sqrt{8}$) of the sample, which is the expected value of V/A (l_f) as $S \rightarrow 1$. We do not know why some of our deduced values of these parameters exceed these limits at high saturation, $S > 0.99$; it may be due to the neglected capillary effects or heterogeneities in the value of the permeability. Notwithstanding, the fact that our analysis comes close to the expected values and gives estimated patch sizes which decrease as the saturation decreases is, we feel, significant substantiation of our approach in this Letter.

Fig. 2

It is clear from this figure that the deduced values of the patch sizes are quite plausible. However, at each saturation, two experimentally determined numbers have been used to deduce the two unknowns, V/A and l_f . As a check, we use our deduced values of V/A and l_f as functions of saturation to calculate $\Re(\tilde{K})(\omega)$ at ultrasonic frequencies, 50 and 100 kHz, and compare it against the measurements of the compressional wave velocities by *Cadoret et al.* [1993, 1995, 1998]. The attenuation was not measured. Fig. 3 shows our findings. There is a reasonable agreement, but one should be cautious since the ultrasonic frequency regime lies barely within the scope of the APS theory, as can be seen from Table I: ω_B lies within the same frequency decade as both 50 kHz and 100 kHz where the quasistatic approximation of the Biot theory starts to break down, ω_x also lies in the same decade, but, as noted above, this is an overestimate for the open interface case and the APS theory should still give a reasonably good approximation even when $\omega \sim \omega_x/2$. Also, by comparing the “flat” regions (marked by dotted lines) in the top and bottom plots, one can observe that the frame bulk modulus, K_b itself scales with frequency. This effect lies outside the scope of the APS theory by invalidating assumption (1). Nevertheless, we report a good agreement between measured and calculated values of $\Re(\tilde{K})$ as seen in Fig. 3.

Fig. 3

The APS theory appears not to apply to the samples with either very high or very low permeability.

First, the rocks with high permeability (Espeil and Saint Pantaleon) have a low Biot frequency (Table I) and even the sonic measurements are separated from ω_B by only a decade in frequency. The inconsistency of the APS theory in this regime can be seen from Fig. 1; the real part of the bulk modulus exceeds K_{BGH} at high saturation in the case of closed interface, but within the APS theory, $\Re(\tilde{K})$ is always less than K_{BGH} . Also, in the case of the open interface, $\Re(\tilde{K})$ has very little variation with saturation, which implies the low frequency regime, but the significant attenuation for $S > 0.8$ contradicts this. As was shown in [Johnson, 2001] for frequencies comparable to ω_B , the attenuation predicted by the full Biot theory can be much larger than that predicted by the APS theory. The theory should not work here, and it doesn't.

Second, the rocks with low permeability (Lavoux RF, Bretigny and Brauvilliers) also have experimental values of the bulk modulus inconsistent with the APS theory; this can be observed in Fig. 1 where we see that the real part of \tilde{K} exceeds K_{BGH} . It is true that the neglected capillary forces may act to pin the water-air interface but, within the context of the Biot theory, the modulus cannot exceed K_{BGH} . Rather, the data are suggestive of a non-Biot mechanism, such as microscopic squirt, though we have no specific evidence for this.

In conclusion, we have shown how the APS theory [Johnson, 2001] of frequency-dependent acoustics in patchy-saturated media can be used to extract information about characteristic fluid patch sizes in partially saturated rocks. We demonstrated this by analyzing resonant bar measurements by Cadoret *et al.* [1993, 1995, 1998] of the acoustic properties of eight types of limestone. For limestones with intermediate values of permeability, the APS theory describes the dominant mechanism of dispersion and attenuation due to patchy-saturation of the sample. Limestones with high values of permeability lie outside the scope of the quasistatic Biot theory and, therefore, invalidate the assumptions of the APS theory, while in the limestones with low permeability, there is another significant mechanism of dispersion and attenuation. Strictly speaking, the ultrasonic measurements of Cadoret lie outside the range of validity of the quasistatic Biot assumption for all eight types of limestone. Nonetheless, we conclude that one can still deduce patch size and shape characteristics of

the saturation pattern from the acoustic measurements, when the assumptions inherent to the APS theory are maintained in the experiments.

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Table 1. Characteristics of the limestones used in the measurements of *Cadoret* [1993].

k regime	Limestone	Porosity, %	k_H^a , mD ^b	k_V^a , mD ^b	$\omega/2\pi^{c,d}$	$\omega_B/2\pi^c$	$\omega_x/2\pi^c$
low	Lavoux RF	24	7.5	8.9	1.3	2400	23
	Bretigny	18	15.0	2.0	1.7	800	30
	Brauwilliers	41	15.4	2.1	1.2	2800	22
medium	Lavoux RG	24	16	44	1.4	400	25
	Estailades	30	255	269	1.2	100	22
	Ménerbes	34	320	288	1.1	99	21
high	Espeil	28	1853	1211	1.1	13	20
	Saint Pantaleon	35	4067	2141	1.1	8.2	21

^a k_H is horizontal and k_V is vertical permeability.

^bOne mD = $10^{-11} cm^2$.

^cFrequencies are in kHz.

^dResonant frequency ω corresponds to the fully-saturated sample.

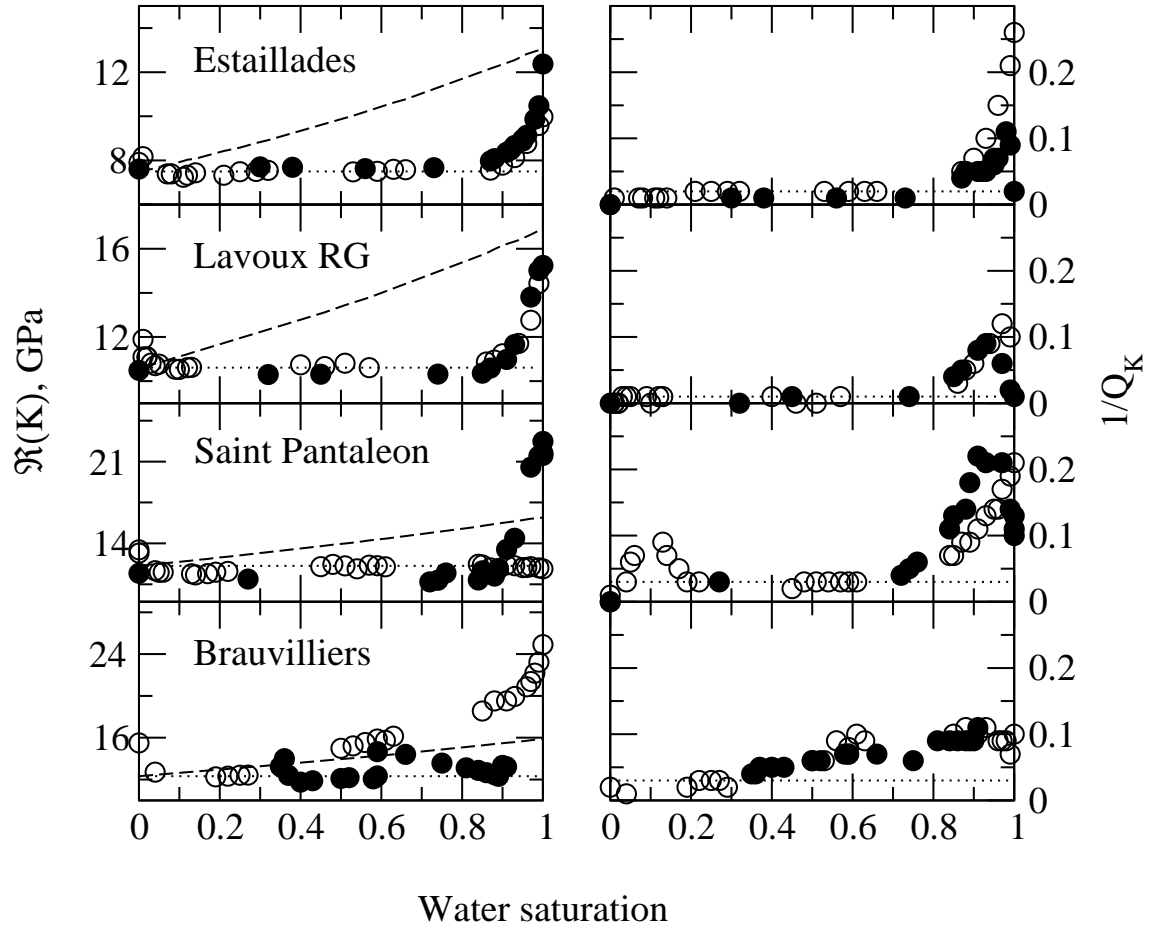


Figure 1. Left plots show dispersion and right plots show attenuation at sonic frequencies of patchy-saturated limestones. Open and filled circles show experimental results which we extract directly from the measurements of *Cadoret* [1993] of the velocities and attenuation of extensional and shear waves. Open (filled) circles correspond to the open (sealed) interface. On the left plots: the dashed line is the high-frequency limit, K_{BGH} , the dotted line is the low-frequency limit, K_{BGW} . On the right plots: the dotted line marks attenuation due to non-patchy-saturation mechanisms. All dashed and dotted lines in the figure correspond to the open interface case.

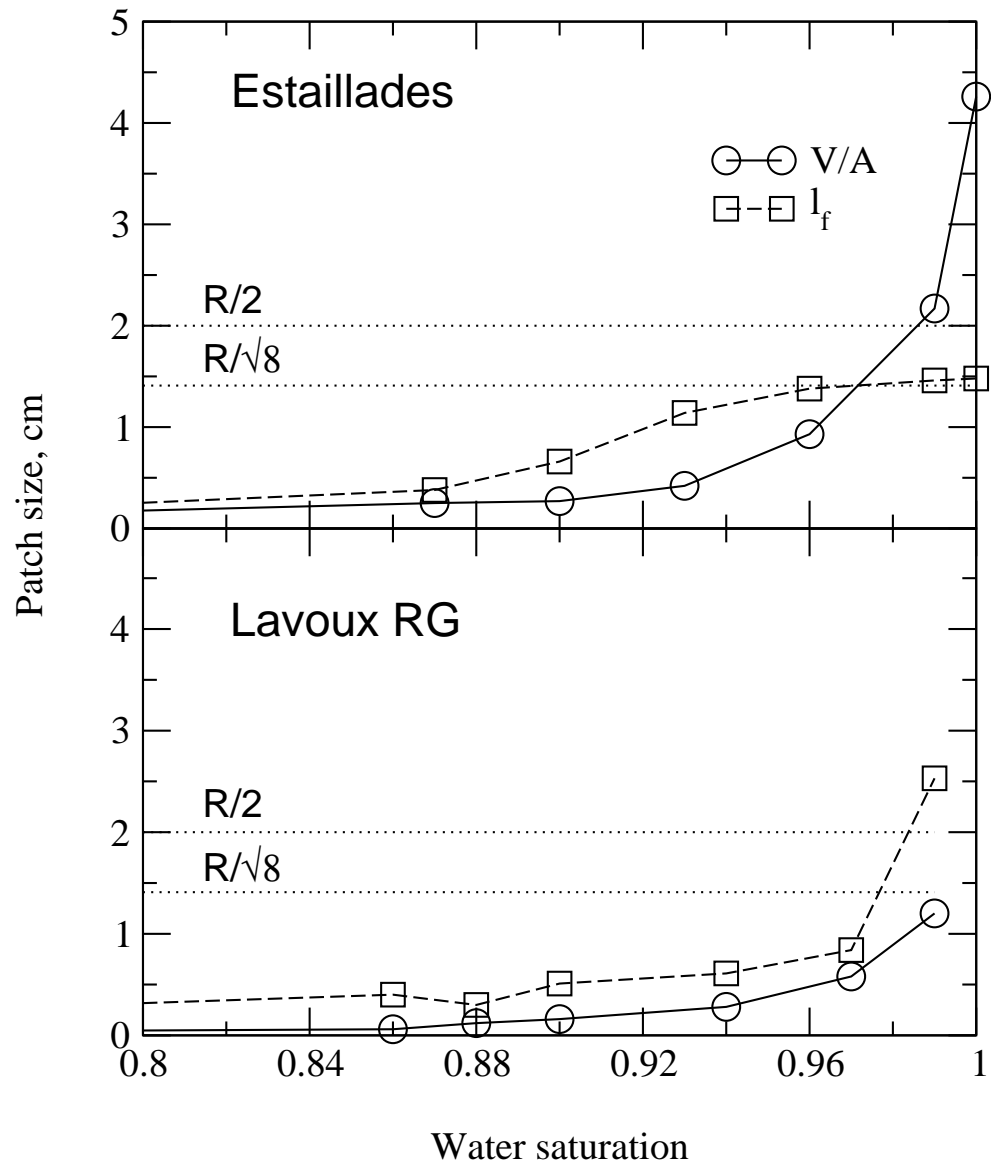


Figure 2. Characteristic water patch sizes calculated from the dispersion and attenuation data in Fig. 1 using the APS theory.

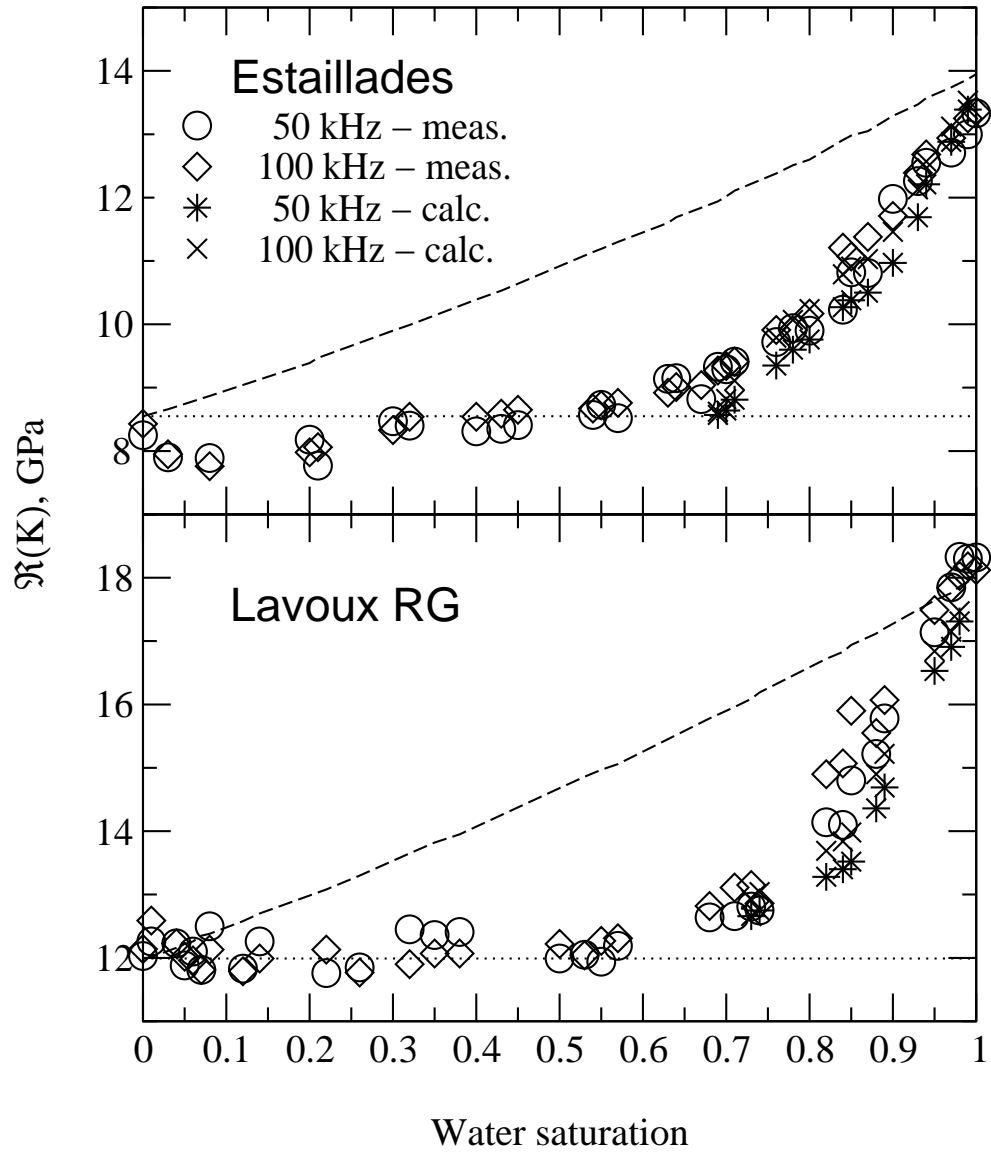


Figure 3. Circles and diamonds show ultrasonic dispersion extracted from Cadoret’s measurements of compressional wave velocities; stars and crosses show corresponding calculated results using parameters from Fig. 2.